Mathematical Analysis of

Royal Baccarat Bonus

Prepared for

New Vision Gaming

By

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Introduction

Royal Baccarat Bonus is a side bet for the game of Baccarat. After the regular Baccarat deal is completed, the dealer forms the best poker hand that can be made with the 4, 5, or 6 cards that were dealt and pays players who made the bonus wager according to a pay table, with a progressive Jackpot for the top award that is split among all players at the table who made the side bet.

Game Rules

The game is to be played at a Baccarat table where the dealer deals only one player hand and one banker hand, to be wagered on by all the players at the table. An 8-deck shoe or a continuous shuffling machine with 8 decks must be used.

Players may wager $1 or more on the side bet. The game may be played either in a "Progressive" configuration, where the top payout is displayed as a dollar amount that is accumulated and then split equally by all the players who made the Progressive bet, and the lower payouts are odds payouts that are proportional to the amount wagered, or in a "Non-Progressive" configuration, where all payouts are proportional to the amount wagered. (The Return To Player and House Edge will be calculated assuming a $1 wager; players who wager more than $1 in the Progressive configuration will receive a lower return and the casino will make a larger House Edge on their bets.)

Baccarat play proceeds as usual after the initial wagers are made. At the end of the round, either 4, 5, or 6 cards will have been exposed in the player hand and banker hand combined. The best poker hand that can be made from the exposed cards determines the bonus payout.

Pay tables for the Progressive configuration are shown on a "for 1" basis. The player's initial wager is not returned. For every dollar wagered, a fixed percentage is set aside and allocated to the Progressive Jackpot. This Jackpot resets to a seed value after it is won, and increases with each Progressive side wager made. Progressive pay tables have a top award for "6 of a kind" or alternatively 6 Card Straight Flush.

Pay tables for the Non-Progressive configuration are shown on a "to 1" basis. The player's initial wager **is** returned. Some pay tables have a top award for "6 of a kind", some for "6 card straight flush", and the others have a top award for (5-card) "Royal Flush".

All other awards involve the best possible 5-card poker hand; if only 4 cards were exposed in the Baccarat game, then some 5-card hands (Straight Flush, Full House, Straight, Flush) are impossible but it is still possible to get Four of a Kind, Three of a Kind, Two Pair, and One Pair. Because of the possibility of getting 6 cards in certain circumstances, a Flush is more likely than a Straight, and a Full House is more likely than both a Straight and a Flush, but the standard Poker ordering is kept so that a Full House pays more than a Flush and a Straight pays less. If more than one kind of winning hand can be formed, players are paid the highest single payout that can be achieved according to the pay table.14

Progressive and 19 Non-Progressive pay tables were evaluated. In the appendix, a variation with a top pay of "6 card Straight Flush" instead of "6 of a Kind" is analyzed.

Mathematical Analysis

A computer program was written to calculate the exact probability of each sequence of 4, 5, or 6 card denominations from an 8-deck shoe, by applying the rules of Baccarat to each of the $13^6 = 4,826,809$ sequences. These were sorted and combined, and the probability of obtaining a flush or a straight flush for each sequence was calculated using combinatorial mathematics. If a 6-card hand included both a 5-card Flush and a 5-card Straight, it was counted as a Flush. The following results were obtained for the overall probabilities of each hand type and the outcomes for each pay table:

Hand type	Probability	1 in
6 of a kind	1.1748E-06	851207.87
5 card RF	5.41553E-06	184654.04
5 card SF	1.82275E-05	54862.25
5 of a kind	7.02367E-05	14237.58
4 of a kind	0.002455579	407.24
5 card straight	0.004820815	207.43
5 card flush	0.006558555	152.47
Full House	0.008021646	124.66
3 of a Kind	0.041200152	24.27
2 Pair	0.07614579	13.13
1 Pair JQKA	0.126220477	7.92
1 Pair 2-10	0.292715463	3.42
0 Pair	0.441766468	2.26

Progressive "For 1"	P1	P2	P3	P4	P5	P6	P7
6 of a kind (seed)	5000	5000	5000	5000	5000	5000	5000
5 card RF	1000	1000	1000	1000	1000	1000	1000
5 card SF	500	500	500	500	500	500	500
5 of a kind	250	200	200	250	250	250	200
4 of a kind	25	25	25	25	30	50	50
Full House	20	20	20	25	25	20	20
5 card flush	15	15	15	20	15	15	15
5 card straight	10	10	10	10	10	10	10
3 of a Kind	3	4	3	3	4	3	3
2 Pair	2	2	2	2	2	2	2
1 Pair JQKA	1	1	1	0	0	0	0
1 Pair 2-10	0	0	0	0	0	0	0
0 Pair	0	0	0	0	0	0	0
Progressive Ctrb	12.000%	12.000%	17.000%	15.000%	15.000%	22.000%	22.000%
Avg Pot (bet units)	107,144.94	107,144.94	149,705.34	132,681.18	132,681.18	192,265.73	192,265.73
House Edge	**7.15%**	**3.38%**	**2.50%**	**9.48%**	**7.42%**	**3.63%**	**3.99%**
Hit frequency	26.55%	26.55%	26.55%	13.93%	13.93%	13.93%	13.93%
Meter seed RTP	0.587%	0.587%	0.587%	0.587%	0.587%	0.587%	0.587%

Progressive "For 1"	P8	P9	P10	P11	P12	P13	P14
6 of a kind (seed)	5000	5000	5000	5000	5000	5000	5000
5 card RF	1000	1000	1000	1000	1000	1000	1000
5 card SF	500	500	500	500	500	500	500
5 of a kind	250	400	200	250	250	250	250
4 of a kind	25	25	25	25	25	25	25
Full House	20	20	20	20	20	20	20
5 card flush	15	15	15	15	15	10	15
5 card straight	10	10	10	10	10	5	5
3 of a Kind	4	4	4	4	3	3	3
2 Pair	2	2	3	2	2	2	2
1 Pair JQKA	0	0	0	0	0	0	0
1 Pair 2-10	0	0	0	0	0	0	0
0 Pair	0	0	0	0	0	0	0
Progressive Ctrb	17.000%	19.000%	17.000%	20.000%	22.000%	20.000%	20.000%
Avg Pot (bet units)	149,705.34	166,729.49	149,705.34	175,241.57	192,265.73	175,241.57	175,241.57
House Edge	**10.65%**	**7.60%**	**3.39%**	**7.65%**	**9.77%**	**17.46%**	**14.18%**
Hit frequency	13.93%	13.93%	13.93%	13.93%	13.93%	13.93%	13.93%
Meter seed RTP	0.587%	0.587%	0.587%	0.587%	0.587%	0.587%	0.587%

Non-Prog"To 1"	N1	N2	N3	N4	N5	N6	N7	N8
5 card RF	5000	5000	5000	5000	5000	5000	5000	5000
5 card SF	1000	1000	1000	1000	1000	1000	1000	1000
5 of a kind	500	500	500	500	500	500	500	500
4 of a kind	20	25	50	50	25	25	50	50
Full House	10	10	20	15	20	15	15	20
5 card flush	5	6	15	10	10	10	10	15
5 card straight	4	5	5	5	5	5	5	5
3 of a Kind	3	3	4	4	4	4	4	3
2 Pair	2	2	2	3	3	3	3	2
1 Pair JQKA	1	1	-1	-1	-1	-1	-1	-1
1 Pair 2-10	-1	-1	-1	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1	-1	-1	-1
House Edge	**7.00%**	**4.63%**	**5.69%**	**5.37%**	**7.49%**	**11.50%**	**5.37%**	**9.81%**
Hit frequency	26.55%	26.55%	13.93%	13.93%	13.93%	13.93%	13.93%	13.93%

Non-Prog"To 1"	N9	N10	N11	N12	N13	N14
6 of a kind	10000	5000	10000	10000	5000	5000
5 card RF	2000	2000	2000	5000	1000	1000
5 card SF	1000	1000	1000	1000	500	500
5 of a kind	500	500	500	500	250	250
4 of a kind	20	20	20	25	20	20
Full House	10	10	15	10	15	10
5 card flush	5	5	5	5	10	8
5 card straight	4	4	4	4	5	5
3 of a Kind	3	3	3	3	3	4
2 Pair	2	2	2	2	2	2
1 Pair JQKA	1	1	1	1	1	1
1 Pair 2-10	-1	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1	-1
House Edge	**7.50%**	**8.09%**	**3.49%**	**4.65%**	**3.53%**	**4.73%**
Hit frequency	26.55%	26.55%	26.55%	26.55%	26.55%	26.55%

Non-Prog"To 1"	N15	N16	N17	N18	N19
6 of a kind	2000	1000	2000	2000	1000
5 card RF	1000	500	500	500	500
5 card SF	500	250	250	400	400
5 of a kind	250	200	200	250	250
4 of a kind	20	20	20	20	20
Full House	10	10	10	10	10
5 card flush	8	8	8	8	8
5 card straight	5	5	5	5	5
3 of a Kind	4	4	4	4	4
2 Pair	2	2	2	2	2
1 Pair JQKA	1	1	1	1	1
1 Pair 2-10	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1
House Edge	**5.08%**	**8.41%**	**8.29%**	**8.29%**	**8.41%**
Hit frequency	26.55%	26.54%	26.54%	26.54%	26.54%

Pay Tables N1 through N19 are all shown above on a "to 1" odds payout basis.

Summary

Royal Baccarat Bonus can be configured with a variety of pay tables and Jackpot types. In the pay tables analyzed here, House Edges ranged between 2.50% and 17.46%. For any Progressive pay table, the House Edge can be precisely controlled by changing the Progressive Contribution percentage. The top award occurs an average of 1 out of 851,208 hands when it is "6 of a kind", and an average of 1 out of 184,654 hands when it is a "5 card Royal Flush".

For the Progressive pay tables, even though the House Edge is always positive, from the player's point of view, the return to player fluctuates depending on the size of Progressive Jackpot. Quite commonly, the Jackpot will be high enough to give the game a return of over 100% if only one player makes the side bet. However, this return declines significantly if multiple players participate, because when the jackpot hits the winnings are split between all players. Players who wager more than the minimum have a lower House Edge for the Progressive pay tables, but the bet size makes no difference to the House Edge for the Non-Progressive pay tables.

The Progressive pay tables shown here have a jackpot seed value of $5,000, when the unit wager is $1. This corresponds to a "seed RTP" of 0.587%, when the Jackpot is triggered by a "6 of a kind". Alternatively, 0.587% of each wager may be accumulated in a secondary pot, which will be used to reseed the jackpot after it is won. In that case, the starting value of the jackpot will AVERAGE $5,000 rather than being $5,000 every time. The overall House Edge is the same whichever way this is done. It is also possible to change the starting value without changing the overall House Edge, by reducing the "progressive contribution". For example, if the starting value is doubled from $5,000 to $10,000 then the "Seed RTP" increases by 0.587% to 1.174%, and the "progressive contribution" should be reduced to compensate. In pay table P1, for example, the progressive contribution would be reduced by 0.587%, from 12.000% to 11.413%, so that the overall House Edge would remain at 7.15%. This could be done either with a fixed seed value of $10,000 or with a secondary pot funded by 1.174% of each wager that would result in an average starting value of $10,000. In all cases the average overall jackpot size would remain $107,145 for that pay table.

For the Non-Progressive pay tables, it is possible to make the top award be a fixed dollar amount rather than a proportional odds payout. This is desirable in order to allow players to make larger wagers without a lucky player who bets $500 and wins the top award costing the casino a huge amount of money.

When the top award is a 5-card royal flush: probability of winning is 0.00000541553, so every change of the ratio of top payout to bet size by 1000 units changes the House Edge in the opposite direction by 0.54%. In pay table N1, the odds payout given is 5000 to 1, and the House Edge is 7.00%, which means a fixed top payout of $25,000 would be a House Edge of 7.00% on a $5 unit bet. When the player bets

$10, the ratio decreases from 5000 to 2500, so the House Edge would be increased by (2.5) (0.54) = 1.35% to 8.35%. With a bet of $25, the ratio decreases from 5000 to 1000, so the House Edge would be increased by (4) (0.54) =2.16% to 9.16%. If a $1 bet is allowed, the ratio increases from 5000 to 25000, so the House Edge is decreased by 20(0.54)=10.8% giving a player advantage of 3.8% -- this shows that the given pay table cannot support both a $25,000 top award and a $1 minimum bet. If the top award were changed from $25,000 to $10,000 while allowing a $1 bet, then the ratio would increase from 5000 to 10000 and the House edge would decrease by 5(0.54)=2.7% to 4.30%.

When the top award is 6 of a kind, the math works the same way, except that the probability of winning is 0.0000011748 so instead of changing by 0.54% for each 1000-unit change in the ratio of award to bet, the House Edge only changes by 0.12%. If a pay table is used with a top award of a 6-card straight flush, which has a probability of 0.000000339847, the House Edge changes by only 0.034% for each 1000-unit change in the ratio.

Appendix

An alternative configuration uses "6 card straight flush" as the top payout rather than "6 of a kind". This award occurs an average of 1 in 2,942,496 hands. Probabilities for the top hands are adjusted as follows:

Hand Type	Probability	1 in
6 card SF	3.39847E-07	2942496.33
5 card RF	5.34505E-06	187089.13
5 card SF	1.79581E-05	55685.15
5 of a kind	7.14115E-05	14003.35

The pay tables are unchanged, but the average jackpot size and House Edge are altered as shown below (pay tables N1 through N8 are unaffected). The "seed RTP" for a $5000 jackpot and a $1 unit wager would be 0.170% (instead of the 0.587% for the original configuration) and the other progressive calculations would change correspondingly.

Pay Table	6oak top HE	6 SF top HE	6oak avg pot	6 SF avg pot
P1	7.15%	7.56%	$107,144.94	$358,099.56
P2	3.38%	3.80%	$107,144.94	$358,099.56
P3	2.50%	2.92%	$149,705.34	$505,224.38
P4	9.48%	9.89%	$132,681.18	$446,374.45
P5	7.42%	7.82%	$132,681.18	$446,374.45
P6	3.63%	4.04%	$192,265.73	$652,349.19
P7	3.99%	4.40%	$192,265.73	$652,349.19
P8	10.65%	11.06%	$149,705.34	$505,224.38
P9	7.60%	7.99%	$166,729.49	$564,074.30
P10	3.39%	3.80%	$149,705.34	$505,224.38
P11	7.65%	8.06%	$175,241.57	$593,499.27
P12	9.77%	10.18%	$192,265.73	$652,349.19
P13	17.46%	17.87%	$175,241.57	$593,499.27
P14	14.18%	14.59%	$175,241.57	$593,499.27
N9	7.50%	8.32%	N/A	N/A
N10	8.09%	8.49%	N/A	N/A
N11	3.49%	4.31%	N/A	N/A
N12	4.65%	5.49%	N/A	N/A
N13	3.53%	3.94%	N/A	N/A
N14	4.73%	5.14%	N/A	N/A
N15	5.08%	5.24%	N/A	N/A
N16	8.21%	8.49%	N/A	N/A
N17	8.49%	8.46%	N/A	N/A
N18	8.49%	8.46%	N/A	N/A
N19	8.21%	8.49%	N/A	N/A

Appendix 2: Fixed Top Payout for non-Progressive "to 1" pay tables N1 through N19

The House Edges shown for Pay tables N1 through N19 assume either a proportional odds payout, or a fixed dollar amount top payout and a $1 bet. If players are allowed to bet $5 on the wager, but the top payout is still restricted, the House Edge changes as follows:

Note that in pay tables N15 through N19, a $5 bet with the proportional odds payout for Royal Flushes and/or Straight Flushes might exceed the fixed payout for 6 of a kind. If the top payout also represents a maximum payout for other awards, the House Edge would be further increased. For example, in pay table N15, if the top payout is capped at $2000, then the 5 card Royal Flush and 5 Card Straight Flush would also only pay $2000 giving effective odds of 400 for 1 or 399 to 1 on those wagers. This would increase the House Edge further, from 5.27% to 5.78%. The effect would be smaller for the other pay tables.

Non-Prog"To 1"	N1	N2	N3	N4	N5	N6	N7	N8
5 card RF	$5000	$5000	$5000	$5000	$5000	$5000	$5000	$5000
5 card SF	1000	1000	1000	1000	1000	1000	1000	1000
5 of a kind	500	500	500	500	500	500	500	500
4 of a kind	20	25	50	50	25	25	50	50
Full House	10	10	20	15	20	15	15	20
5 card flush	5	6	15	10	10	10	10	15
5 card straight	4	5	5	5	5	5	5	5
3 of a Kind	3	3	4	4	4	4	4	3
2 Pair	2	2	2	3	3	3	3	2
1 Pair JQKA	1	1	-1	-1	-1	-1	-1	-1
1 Pair 2-10	-1	-1	-1	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1	-1	-1	-1
House Edge $5 bet	**9.16%**	**6.79%**	**7.85%**	**7.53%**	**9.65%**	**13.66%**	**7.53%**	**11.97%**
Hit frequency	26.55%	26.55%	13.93%	13.93%	13.93%	13.93%	13.93%	13.93%

Non-Prog"To 1"	N9	N10	N11	N12	N13	N14
6 of a kind	$10000	$5000	$10000	$10000	$5000	$5000
5 card RF	2000	2000	2000	5000	1000	1000
5 card SF	1000	1000	1000	1000	500	500
5 of a kind	500	500	500	500	250	250
4 of a kind	20	20	20	25	20	20
Full House	10	10	15	10	15	10
5 card flush	5	5	5	5	10	8
5 card straight	4	4	4	4	5	5
3 of a Kind	3	3	3	3	3	4
2 Pair	2	2	2	2	2	2
1 Pair JQKA	1	1	1	1	1	1
1 Pair 2-10	-1	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1	-1
House Edge $5 bet	**8.46%**	**8.57%**	**4.45%**	**5.61%**	**4.01%**	**5.21%**
Hit frequency	26.55%	26.55%	26.55%	26.55%	26.55%	26.55%

Non-Prog"To 1"	N15	N16	N17	N18	N19
6 of a kind	$2000	$1000	$2000	$2000	$1000
5 card RF	1000	500	500	500	500
5 card SF	500	250	250	400	400
5 of a kind	250	200	200	250	250
4 of a kind	20	20	20	20	20
Full House	10	10	10	10	10
5 card flush	8	8	8	8	8
5 card straight	5	5	5	5	5
3 of a Kind	4	4	4	4	4
2 Pair	2	2	2	2	2
1 Pair JQKA	1	1	1	1	1
1 Pair 2-10	-1	-1	-1	-1	-1
0 Pair	-1	-1	-1	-1	-1
House Edge $5 bet	**5.27%**	**8.51%**	**8.48%**	**8.48%**	**8.51%**
Hit frequency	26.55%	26.54%	26.54%	26.54%	26.54%