Mathematical analysis of Jackpot 21, Jackpot 21 Progressive Double Twist Blackjack Prepared for New Vision Gaming

## By

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# Analysis of Jackpot 21

#### Introduction

Jackpot 21 is a Blackjack side bet based on the 5-card Poker hand formed by the dealer's one up card and the first two cards of two player Blackjack hands, which wins if the 5-card Poker hand achieves a Pair of Jacks or better. The game may be played with a variety of pay tables. An alternative version based on the player's 4-card Poker hand formed only by the player's Blackjack hands, or by the player's first two cards and either the dealer's first two cards or the dealer's upcard and a special community card, which wins if the player's 4-card hand has a pair of Sixes or better, is analyzed in the Appendix to this report.

#### **Game Rules**

#### Standard Blackjack Rules Will Apply.

To start the game player(s) must place a wager on one or both of the blackjack circles, this will determine which blackjack hand(s) the player is wagering on. The dealer shall deal two cards face up to each blackjack spot that has a blackjack wager, and one card face up and one card face down to herself. The bonus wager is optional, should a player decide to participate in the bonus they must place wagers on both blackjack hands then place their bonus wager in the circle marked Jackpot 21.

To win a bonus payout players must achieve a pair of Jacks or higher by using their two initial blackjack hands totaling 4 cards with the dealer's 1 up card for a total of 5 cards. Player(s) are paid based on the highest single payout for a poker hand they achieve according to a pay table. To participate in the bonus players must place their Jackpot 21 wagers before any cards are dealt.

Before any other bets are settled and before any other cards are dealt, the dealer evaluates the player's 5-card poker hand formed by the 1 dealer up card and the 4 player cards for each player who made the Jackpot 21 bonus wager. Poker hands which have a pair of Jacks or better win and are paid according to a pay table. In the alternative version of the game treated in the Appendix, only the 4 player cards are used and player hands with a pair of Sixes or better win.

Poker hands which do not have at least a pair of Jacks lose their Jackpot 21 bonus wager. (Optionally, the casino may choose to allow players who only bet on one Blackjack hand to make the Jackpot 21 bonus wager, player(s) are then dealt two additional cards solely for the purpose of settling that wager, rather than requiring that a player play two separate Blackjack hands.)

Note: Because this game is played at a Blackjack table with a multi-deck shoe, 5-of-a-kinds are possible, and it is also possible for hands which contain pairs, trips, and quads to also be flushes. 5-of-a-kinds, 4-of-a-kinds, and Full Houses which are also flushes will be counted as the higher hand rather than as flushes. Flushes which are also 3-of-a-kinds, 2-pair hands, or 1-pair hands will be counted as flushes rather than as the lower hands.

The following pay tables are intended for different sizes of shoe because the hand probabilities depend strongly on how many decks are in the shoe. Odds pays are shown on a "To 1" basis: for example, a high pair of Jacks or Better is paid 1:1 or even money. Alternative 4-card pay tables, based on the player 4 cards only, or on the two cards of a player hand and either the dealer's first two cards or the dealer's upcard and a special community card, are given in the Appendix.

|                       | 1 Deck-A | 1 Deck-B | 2 Deck-A | 2 Deck-B | 4 Deck-A | 4 Deck-B | 6 Deck-A |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|
|                       |          |          |          |          |          |          |          |
| Royal Flush           | 1000     | 2000     | 1000     | 2000     | 1000     | 2000     | 1000     |
| Straight Flush        | 500      | 500      | 500      | 500      | 500      | 500      | 500      |
| Five of a Kind        | N/A      | N/A      | 250      | 500      | 200      | 200      | 250      |
| Four of a Kind        | 50       | 200      | 50       | 50       | 30       | 50       | 50       |
| Full House            | 25       | 50       | 25       | 20       | 15       | 20       | 25       |
| Flush (no 4k5kFH)     | 20       | 25       | 15       | 15       | 15       | 15       | 15       |
| Straight              | 15       | 15       | 10       | 10       | 15       | 10       | 10       |
| 3 of a Kind (noFlush) | 5        | 10       | 5        | 5        | 5        | 5        | 3        |
| Two Pair (no Flush)   | 3        | 3        | 3        | 3        | 2        | 2        | 2        |
| High Pair (no Flush)  | 2        | 1        | 1        | 1        | 1        | 1        | 1        |

|                       | 6 Deck-B | 8 Deck-A | 8 Deck-B | 8 Deck-C | 8 Deck-D | 8 Deck-E |
|-----------------------|----------|----------|----------|----------|----------|----------|
|                       |          |          |          |          |          |          |
| Royal Flush           | 2000     | 1000     | 5000     | 500      | 2000     | 2000     |
| Straight Flush        | 500      | 250      | 500      | 250      | 500      | 500      |
| Five of a Kind        | 250      | 200      | 250      | 200      | 200      | 250      |
| Four of a Kind        | 50       | 50       | 50       | 50       | 50       | 50       |
| Full House            | 25       | 25       | 25       | 20       | 25       | 25       |
| Flush (no 4k5kFH)     | 15       | 15       | 15       | 15       | 20       | 15       |
| Straight              | 10       | 10       | 10       | 10       | 10       | 10       |
| 3 of a Kind (noFlush) | 3        | 3        | 3        | 3        | 3        | 3        |
| Two Pair (no Flush)   | 2        | 2        | 2        | 2        | 2        | 2        |
| High Pair (no Flush)  | 1        | 1        | 1        | 1        | 1        | 1        |

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### Analysis

This game was analyzed by using combinatorial mathematics and probability. The following probabilities were obtained for each hand type and shoe size:

| Number of Decks        | 1          | 2          | 4          | 6           | 8            |
|------------------------|------------|------------|------------|-------------|--------------|
| Diamond Royal          | 0.0000038  | 0.0000035  | 0.0000033  | 0.0000033   | 0.0000032    |
| Royal Flush            | 0.00000115 | 0.00000104 | 0.0000099  | 0.0000098   | 0.00000097   |
| Straight Flush         | 0.00001385 | 0.00001253 | 0.00001193 | 0.00001173  | 0.00001164   |
| Five of a Kind         | 0.00000000 | 0.00000792 | 0.00001837 | 0.00002316  | 0.00002583   |
| Four of a Kind         | 0.00024010 | 0.00094995 | 0.00146964 | 0.00166764  | 0.00177128   |
| Full House             | 0.00144058 | 0.00265987 | 0.00339149 | 0.00365291  | 0.00378687   |
| Flush (no 4k5kFH)      | 0.00196540 | 0.00284725 | 0.00334432 | 0.00351644  | 0.00360357   |
| Straight               | 0.00392465 | 0.00354927 | 0.00337906 | 0.00332469  | 0.00329794   |
| 3 of a Kind (no Flush) | 0.02112845 | 0.03343832 | 0.03972239 | 0.04182553  | 0.04287791   |
| Two Pair (no Flush)    | 0.04753902 | 0.05844242 | 0.06379388 | 0.06555826  | 0.06643648   |
| High Pair (no Flush)   | 0.13002124 | 0.13687663 | 0.13949572 | 0.14026301  | 0.14062790   |
| Total deals            | 2598960    | 91962520   | 3091033296 | 23856384552 | 101346274848 |

The House Edge for each pay table is as follows:

| 1 Deck A: 13.08% 1 Deck B: 7.17%% |                  |                  |  |  |
|-----------------------------------|------------------|------------------|--|--|
| 2 Decks A: 8.00%                  | 2 Decks B: 8.99% |                  |  |  |
| 4 Decks A: 7.29%                  | 4 Decks B: 4.21% |                  |  |  |
| 6 Decks A: 6.96%                  | 6 Decks B: 6.83% |                  |  |  |
| 8 Decks-A: 5.58%                  | 8 Decks-B: 4.64% | 8 Decks-C: 7.54% |  |  |
| 8 Decks-D: 3.36%                  | 8 Decks-E: 5.03% |                  |  |  |

### **APPENDIX: 4-Card Pay Tables for Jackpot 21**

Jackpot 21 can also be played with a 4-card pay table in three ways: using only the 4 initial cards from the player's two Blackjack hands, or using the initial two cards from a player hand combined with either the dealer's first two cards or the dealer's upcard and a special community card which is dealt if any players have made the side wager. In the second variation, bets cannot be settled until after players have completed their hands and the dealer's upcard has been turned over. In the third variation, the dealer must deal an additional face-up "community card" after dealing his own cards, if any players have made the 4-card side bet. Four pay tables were analyzed, where all hands containing a pair of Sixes or better are winners.

The calculated probabilities for all 4-card hands are as follows:

| Probabilities       | 6 decks   | 8 decks   |
|---------------------|-----------|-----------|
| 4 of a kind suited  | 0.0000020 | 0.0000030 |
| 4 card royal flush  | 0.0000134 | 0.0000133 |
| straight flush      | 0.0001339 | 0.0001332 |
| 4 of a kind         | 0.0003547 | 0.0003771 |
| straight            | 0.0092763 | 0.0092315 |
| 2 pair flush        | 0.0001813 | 0.0001989 |
| 3 of a kind flush   | 0.0001933 | 0.0002273 |
| 1 pair flush        | 0.0047854 | 0.0050004 |
| flush               | 0.0094235 | 0.0093780 |
| 2 pair              | 0.0151610 | 0.0154030 |
| 3 of a kind         | 0.0193736 | 0.0199042 |
| High Pair (6's-A's) | 0.2405209 | 0.2418325 |
| Low Pair (2's-5's)  | 0.1068982 | 0.1074811 |
| no pair             | 0.5936826 | 0.5908164 |
| Total               | 1         | 1         |

The following results were obtained (all payouts are on a "to 1" basis: pairs of Sixes through Aces are paid even money and pairs of Deuces through Fives lose):

|                    | 6 Decks-<br>A | 8 Decks-<br>A | 6 Decks-<br>B | 8 Decks-<br>B |
|--------------------|---------------|---------------|---------------|---------------|
| 4 of a kind suited | 1000          | 1000          | 500           | 1000          |
| 4 card royal flush | 500           | 500           | 250           | 250           |
| straight flush     | 200           | 200           | 100           | 100           |
| 4 of a kind        | 50            | 50            | 50            | 50            |
| straight           | 20            | 10            | 20            | 15            |
| flush              | 5             | 10            | 4             | 10            |

| 2 pair              | 4     | 4     | 3     | 3     |
|---------------------|-------|-------|-------|-------|
| 3 of a kind         | 3     | 3     | 2     | 2     |
| High Pair (6's-A's) | 1     | 1     | 1     | 1     |
| Low Pair (2's-5's)  | -1    | -1    | -1    | -1    |
| no pair             | -1    | -1    | -1    | -1    |
| House Edge          | 2.96% | 3.97% | 9.65% | 4.55% |

## **Analysis of Jackpot 21 Progressive**

#### Introduction

Jackpot 21 Progressive is a Blackjack side bet based on the 5-card Poker hand formed by the dealer's one up card and the first two cards of two player Blackjack hands, which wins if the 5-card Poker hand achieves a Pair of Jacks or better. The game may be played with a variety of pay tables. An alternative version based on the player's 4-card Poker hand formed only by the player's Blackjack hands, or by the player's first two cards and either the dealer's first two cards or the dealer's upcard and a special community card, which wins if the player's 4-card hand has a pair of Sixes or better, is analyzed in the Appendix to this report.

#### **Game Rules**

#### Standard Blackjack Rules Will Apply.

To start the game player(s) must place a wager in one or both of the blackjack circles, this will determine which blackjack hand the player is wagering on. The dealer shall deal two cards face up to each blackjack spot that has a blackjack wager, and one card face up and one card face down to herself. The bonus wager is optional, should a player decide to participate in the bonus they must place wagers on both blackjack hands then place their Jackpot 21 Progressive bonus wager in the circle marked Jackpot 21.

To win a bonus payout players must achieve a pair of Jacks or higher by using their two initial blackjack hands totaling 4 cards with the dealer's 1 up card for a total of 5 cards. Player(s) are paid based on the highest single payout for a poker hand they achieve according to a pay table. To participate in the progressive bonus, players must place their Jackpot 21 Progressive bonus wager before any cards are dealt.

Before any other bets are settled and before any other cards are dealt, the dealer evaluates the player's 5 card poker hand, formed by 1 dealer up card and the 4 player cards for each player who made the Jackpot Progressive bonus wager. Poker hands which have a pair of Jacks or better win and are paid according to a pay table. In the alternative version of the game treated in the Appendix, only the 4 player cards are used and player hands with a pair of Sixes or better win.

Poker hands which do not have at least a pair of Jacks lose their Jackpot 21 Progressive bonus wager. (Optionally, the casino may choose to allow players who only bet on one Blackjack hand to make the Jackpot 21 Progressive bonus wager and be dealt an additional two cards solely for the purpose of settling that progressive bonus wager, rather than requiring that a player play two separate Blackjack hands.)

Note: Because this game is played at a Blackjack table with a multi-deck shoe, 5-of-a-kinds are possible, and it is also possible for hands which contain pairs, trips, and quads to also be flushes. 5-of-a-kinds, 4-of-a-kinds, and Full Houses which are also flushes will be counted as the higher hand rather than as flushes. Flushes which are also 3-of-a-kinds, 2-pair hands, or 1-pair hands will be counted as flushes rather than as the lower hands.

This game with a Progressive Jackpot uses pay tables with odds shown on a "for 1" basis instead of a "to 1" basis, because all payouts will be shown as dollar amounts to match the accumulating Jackpot. The Jackpot begins at a fixed "Seed" amount (\$100,000, \$50,000, \$25,000 or \$10,000) and grows until a player hits a Royal Flush. At that point, if the player has a Royal Flush in Diamonds, the entire Jackpot is paid out, and a new one starts at the seed value. If the player has a Royal Flush in Clubs, Hearts, or Spades, then 10% of the Jackpot is paid out to the player and it continues accumulating from the lower value.

The Progressive Contribution is set at 5%, (\$0.05 for each \$1 bet, or \$0.25 for each \$5 bet). The following Pay Tables are intended for different sizes of shoe because the hand probabilities depend strongly on how many decks are in the shoe ("for 1" basis). Alternative 4-card pay tables, based on the player 4 cards only, or on the two cards of a player hand and either the dealer's first two cards or the dealer's upcard and a special community card, are given in the Appendix.

| For 1 Basis with Jackpot | 1DeckA | 1DeckB | 2DeckA | 2DeckB | 4DeckA | 4DeckB |
|--------------------------|--------|--------|--------|--------|--------|--------|
| Diamond Royal            | 100%   | 100%   | 100%   | 100%   | 100%   | 100%   |
| Royal Flush              | 10%    | 10%    | 10%    | 10%    | 10%    | 10%    |
| Straight Flush           | 1500   | 500    | 5000   | 500    | 2500   | 500    |
| Five of a Kind           | N/A    | N/A    | 500    | 400    | 200    | 400    |
| Four of a Kind           | 200    | 200    | 50     | 100    | 30     | 100    |
| Full House               | 50     | 50     | 20     | 25     | 15     | 25     |
| Flush (no 4k5kFH)        | 25     | 25     | 20     | 20     | 15     | 20     |
| Straight                 | 10     | 15     | 8      | 10     | 15     | 10     |
| 3 of a Kind (noFlush)    | 6      | 10     | 4      | 4      | 4      | 5      |
| Two Pair (no Flush)      | 4      | 3      | 3      | 3      | 3      | 3      |
| High Pair (no Flush)     | 2      | 2      | 2      | 2      | 2      | 1      |
|                          |        |        |        |        |        |        |

| For 1 Basis with Jackpot | 6DeckA | 6DeckB | 8DeckA | 8DeckB | 8DeckC | 8DeckD | 8DeckE |
|--------------------------|--------|--------|--------|--------|--------|--------|--------|
| Diamond Royal            | 100%   | 100%   | 100%   | 100%   | 100%   | 100%   | 100%   |
| Royal Flush              | 10%    | 10%    | 10%    | 10%    | 10%    | 10%    | 10%    |
| Straight Flush           | 1000   | 500    | 1000   | 1000   | 500    | 500    | 500    |
| Five of a Kind           | 200    | 400    | 200    | 200    | 250    | 400    | 400    |
| Four of a Kind           | 25     | 50     | 25     | 30     | 50     | 50     | 50     |
| Full House               | 15     | 25     | 15     | 20     | 25     | 25     | 20     |
| Flush (no 4k5kFH)        | 15     | 20     | 15     | 20     | 15     | 20     | 15     |
| Straight                 | 15     | 10     | 15     | 20     | 10     | 10     | 10     |
| 3 of a Kind (noFlush)    | 3      | 5      | 3      | 5      | 5      | 5      | 5      |
| Two Pair (no Flush)      | 3      | 3      | 3      | 3      | 3      | 3      | 3      |
| High Pair (no Flush)     | 2      | 1      | 2      | 1      | 1      | 1      | 1      |

Because this table is to be interpreted as "for 1" odds, a pair of Jacks or Better pays even money (2 for 1) in some pay tables. In the remaining pay tables that hand is a push (1 for 1).

Note that the tables above work with various bet sizes. A minimum bet (typically \$1 or \$5) is necessary to be eligible to win the Progressive Jackpot. All non-jackpot payouts (Straight Flush and below) are on a "for 1 basis". Suppose, for example, that the Jackpot Seed is set at \$100,000 and the minimum bet is \$1. Then a player who bets more than \$1 will receive proportionate odds payouts on non-Jackpot hands, but will receive the same accumulated Jackpot amount on a Royal Flush whether he bets \$1 or \$10. A \$10 bet in 8 Deck Pay Table A, which is listed at 1000 for 1 odds, will receive \$10,000 back on a Straight Flush, while a \$1 bet will receive \$1,000 back on a Straight Flush, but both bets would receive the same Jackpot amount on a Royal Flush. This means that, technically, the odds are better for the player who makes the minimum bet, and the House Edge is larger for players who make larger bets.

#### Analysis

This game was analyzed by using combinatorial mathematics and probability. The following probabilities were obtained for each hand type and shoe size:

| Number of Decks        | 1          | 2          | 4          | 6           | 8            |
|------------------------|------------|------------|------------|-------------|--------------|
| Diamond Royal          | 0.0000038  | 0.0000035  | 0.0000033  | 0.0000033   | 0.0000032    |
| Royal Flush            | 0.00000115 | 0.00000104 | 0.0000099  | 0.0000098   | 0.0000097    |
| Straight Flush         | 0.00001385 | 0.00001253 | 0.00001193 | 0.00001173  | 0.00001164   |
| Five of a Kind         | 0.00000000 | 0.00000792 | 0.00001837 | 0.00002316  | 0.00002583   |
| Four of a Kind         | 0.00024010 | 0.00094995 | 0.00146964 | 0.00166764  | 0.00177128   |
| Full House             | 0.00144058 | 0.00265987 | 0.00339149 | 0.00365291  | 0.00378687   |
| Flush (no 4k5kFH)      | 0.00196540 | 0.00284725 | 0.00334432 | 0.00351644  | 0.00360357   |
| Straight               | 0.00392465 | 0.00354927 | 0.00337906 | 0.00332469  | 0.00329794   |
| 3 of a Kind (no Flush) | 0.02112845 | 0.03343832 | 0.03972239 | 0.04182553  | 0.04287791   |
| Two Pair (no Flush)    | 0.04753902 | 0.05844242 | 0.06379388 | 0.06555826  | 0.06643648   |
| High Pair (no Flush)   | 0.13002124 | 0.13687663 | 0.13949572 | 0.14026301  | 0.14062790   |
| Total deals            | 2598960    | 91962520   | 3091033296 | 23856384552 | 101346274848 |

The House Edge for each pay table is as follows:

With \$100,000 Progressive, \$1 bet, 5% contribution, pay tables on "for 1" basis:

| 1 Deck-A: 10.53% | 1 Deck-B: 6.26%  |                  |
|------------------|------------------|------------------|
| 2 Decks-A: 7.97% | 2 Decks-B: 6.90% |                  |
| 4 Decks-A: 5.83% | 4 Decks-B: 4.16% |                  |
| 6 Decks-A: 9.93% | 6 Decks-B: 7.79% |                  |
| 8 Decks-A: 8.71% | 8 Decks-B: 7.96% | 8 Decks-C: 8.08% |
| 8 Decks-D: 5.89% | 8 Decks-E: 9.59% |                  |

Adjustment for alternate configurations with smaller Jackpots or larger minimum bets:

\$50,000 seed value, \$1 bet:

| 8 decks: add 1.62% to House Edge (for example, 8-dec | k pay table A has a House Edge of 8.71%+1.62%=10.33%) |
|--|---|
|  |   |

| 6 Decks: add 1.63% | 4 Decks: add 1.66% | 2 Decks: add 1.74% | 1 Deck: add 1.92% |
|--------------------|--------------------|--------------------|-------------------|
|                    |                    |                    |                   |

\$100,000 seed value, \$2 bet: same -- House Edge depends on ratio of Jackpot seed to minimum bet, not on their absolute amounts.

\$25,000 seed value, \$1 bet:

8 decks: add 2.42% to House Edge (for example, 8-deck pay table A has a House Edge of 8.71%+2.42%=11.13%)

6 Decks: add 2.44% 4 Decks: add 2.48% 2 Decks: add 2.61% 1 Deck: add 2.89%

\$50,000 seed value, \$2 bet: same -- House Edge depends on ratio of Jackpot seed to minimum bet, not on their absolute amounts.

\$20,000 seed value, \$1 bet:

8 decks: add 2.58% to House Edge (for example, 8-deck pay table A has a House Edge of 8.71%+2.58%=11.29%)

6 Decks: add 2.60% 4 Decks: add 2.65% 2 Decks: add 2.78% 1 Deck: add 3.08%

\$100,000 seed value, \$5 bet: same -- House Edge depends on ratio of Jackpot seed to minimum bet, not on their absolute amounts.

\$10,000 seed value, \$1 bet:

8 decks: add 2.91% to House Edge (for example, 8-deck pay table A has a House Edge of 8.71%+2.91%=11.62%)

6 Decks: add 2.93% 4 Decks: add 2.98% 2 Decks: add 3.13% 1 Deck: add 3.46%

\$20,000 seed value, \$2 bet: same -- House Edge depends on ratio of Jackpot seed to minimum bet, not on their absolute amounts.

\$50,000 seed value, \$5 bet: same -- House Edge depends on ratio of Jackpot seed to minimum bet, not on their absolute amount.

### **APPENDIX: 4-Card Pay Tables for Jackpot 21 Progressive**

Jackpot 21 Progressive can also be played using only the 4 initial cards from the player's two Blackjack hands, or using the initial two cards from a player hand combined with either the dealer's first two cards or the dealer's upcard and a special community card which is dealt if any players have made the side wager. In the second variation, bets cannot be settled until after players have completed their hands and the dealer's upcard has been turned over. In the third variation, the dealer must deal an additional face-up "community card" after dealing his own cards, if any players have made the 4-card side bet. Five pay tables were analyzed, where all hands containing a pair of Sixes or better are winners.

The calculated probabilities for all 4-card hands are as follows:

| Probabilities       | 6 decks   | 8 decks   |
|---------------------|-----------|-----------|
| 4 of a kind suited  | 0.0000020 | 0.0000030 |
| 4 card royal flush  | 0.0000134 | 0.0000133 |
| straight flush      | 0.0001339 | 0.0001332 |
| 4 of a kind         | 0.0003547 | 0.0003771 |
| straight            | 0.0092763 | 0.0092315 |
| 2 pair flush        | 0.0001813 | 0.0001989 |
| 3 of a kind flush   | 0.0001933 | 0.0002273 |
| 1 pair flush        | 0.0047854 | 0.0050004 |
| flush               | 0.0094235 | 0.0093780 |
| 2 pair              | 0.0151610 | 0.0154030 |
| 3 of a kind         | 0.0193736 | 0.0199042 |
| High Pair (6's-A's) | 0.2405209 | 0.2418325 |
| Low Pair (2's-5's)  | 0.1068982 | 0.1074811 |
| no pair             | 0.5936826 | 0.5908164 |
| Total               | 1         | 1         |

The following results were obtained (all payouts are on a "to 1" basis: pairs of Sixes through Aces are paid even money and pairs of Deuces through Fives lose):

|                    | 6 Decks- | 6 Decks- | 8 Decks- | 8 Decks- | 8 Decks- |
|--------------------|----------|----------|----------|----------|----------|
|                    | А        | В        | А        | В        | С        |
| 4 of a kind suited | 100%     | 100%     | 100%     | 100%     | 100%     |
| 4 card royal flush | 1000     | 1000     | 500      | 1000     | 1000     |
| straight flush     | 100      | 100      | 100      | 100      | 100      |
| 4 of a kind        | 100      | 50       | 50       | 50       | 50       |
| straight           | 10       | 10       | 10       | 10       | 10       |
| flush              | 5        | 5        | 5        | 5        | 5        |
| 2 pair             | 4        | 3        | 4        | 4        | 4        |

| 3 of a kind              | 2     | 2     | 2     | 2             | 3             |
|--------------------------|-------|-------|-------|---------------|---------------|
| High Pair (6's-A's)      | 1     | 1     | 1     | 1             | 1             |
| Low Pair (2's-5's)       | -1    | -1    | -1    | -1            | -1            |
| no pair                  | -1    | -1    | -1    | -1            | -1            |
| Jackpot Seed Value       | 10000 | 10000 | 5000  | 10000         | 10000         |
| Progressive Contribution | 5.00% | 5.00% | 5.00% | 5.00 <b>%</b> | 5.00 <b>%</b> |
| House Edge               | 6.26% | 9.55% | 8.51% | 6.36%         | 4.37%         |

The Jackpot Seed Value shown is denominated in betting units. When the minimum bet is \$5, a seed value of 10000 units will reset to \$50,000, and so on.

## **Analysis of Double Twist Blackjack**

#### Introduction

*Double Twist Blackjack* is a Blackjack side bet based on the 3-card Poker hands formed by the dealer's one up card and the first two cards of a player's blackjack hand. Players win if their 3-card hand achieves a poker ranking of a Pair or better. The Table Layout allows players to wager on two separate blackjack hands and two separate three card bonuses. The game may be played with a variety of pay tables. An alternative version based on the player's 4card Poker hand formed only by the player's Blackjack hands, or by the player's first two cards and either the dealer's first two cards or the dealer's upcard and a special community card, which wins if the player's 4-card hand has a pair of Sixes or better, is analyzed in the Appendix to this report.

#### **Game Rules**

#### Standard Blackjack Rules Will Apply.

To start the game players must place a wager in one or both of the blackjack circles, this will determine which blackjack hand the player is wagering on. The dealer shall deal two cards face up to each blackjack spot that has a blackjack wager, and one card face up and one card face down to herself.

The bonus wager is optional, players who have placed a wager on either one of the blackjack hands have the option of making a bonus wager that correlates to that blackjack hand by placing a bonus wager above the blackjack circle in the circle marked Blackjack Bonus.

To win a bonus payout players must achieve a Pair or better by using their initial two cards with the dealer's one up card for a total of 3 cards. Player(s) are paid based on the highest single payout for a 3-card poker hand they achieve according to a pay table. Players lose their bonus wager(s) if their 3 card poker hand is less than a Pair. In the alternative version of the game treated in the Appendix, only the 4 player cards are used and player hands with a pair of Sixes or better win.

Note: Because this game is played at a Blackjack table with a multi-deck shoe, it is also possible for hands which contain pairs or trips to also be flushes. 3-of-a-kinds which are also flushes may be counted as a "3 of a kind suited" hand with a special payout, or may be counted simply as 3 of a kind. Flushes which are also Pair hands will be counted as flushes rather than as the lower hands.

The following pay tables are intended for different sizes of shoe because the hand probabilities depend strongly on how many decks are in the shoe. Odds pays are shown on a "To 1" basis: for example, a Pair is paid 1:1 or even money. Alternative 4-card pay tables, based on the player 4 cards only, or on the two cards of a player hand and either the dealer's first two cards or the dealer's upcard and a special community card, are given in the Appendix.

Nine pay tables are used in which 3 of a kind flushes are treated as 3 of a kinds, and pair flushes are treated as flushes:

| To 1 Basis         | 1DeckA | 1DeckB | 1DeckC | 1DeckD | 6DecksA | 6DecksB | 8DecksA | 8DecksB | 8DecksC |
|--------------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| 3 card Royal Flush | 100    | 100    | 50     | 100    | 50      | 100     | 50      | 100     | 40      |
| Straight Flush     | 50     | 40     | 40     | 40     | 30      | 30      | 30      | 30      | 30      |
| 3 of a Kind        | 30     | 30     | 30     | 30     | 20      | 20      | 20      | 20      | 20      |
| Straight           | 5      | 5      | 6      | 6      | 4       | 4       | 4       | 4       | 4       |
| Flush              | 4      | 4      | 3      | 3      | 3       | 3       | 3       | 3       | 3       |
| Pair               | 1      | 1      | 1      | 1      | 1       | 1       | 1       | 1       | 1       |

Five additional multi-deck pay tables are considered where "3 of a Kind Suited" has its own payout:

| To 1 Basis         | 4DecksA | 6DecksC | 8DecksD | 8DecksE | 8DecksF |
|--------------------|---------|---------|---------|---------|---------|
| 3 card Royal Flush | 50      | 50      | 100     | 50      | 50      |
| 3 of a Kind Suited | 40      | 40      | 50      | 35      | 40      |
| Straight Flush     | 30      | 30      | 25      | 30      | 25      |
| 3 of a Kind        | 25      | 20      | 20      | 20      | 20      |
| Straight           | 4       | 4       | 4       | 4       | 4       |
| Flush              | 3       | 3       | 3       | 3       | 3       |
| Pair               | 1       | 1       | 1       | 1       | 1       |
|                    |         |         |         |         |         |

#### Analysis

This game was analyzed by using combinatorial mathematics and probability. The following probabilities were obtained for each hand type and shoe size:

| Num Decks   | 1               | 2           | 4           | 6           | 8           |
|---|-----------------|-------------|-------------|-------------|-------------|
| 3 Card Royal Flush  | 0.000180995     | 0.000175724 | 0.000173177 | 0.000172341 | 0.000171925 |
| Straight Flush  | 0.00199095      | 0.001932961 | 0.001904947 | 0.00189575  | 0.001891177 |
| 3 of a Kind (incl flush)  | 0.002352941     | 0.003997716 | 0.004924722 | 0.005248418 | 0.005412956 |
| Straight  | 0.032579186     | 0.031630277 | 0.031171867 | 0.031021359 | 0.030946529 |
| Flush (pair, no 3oak)   | 0.04959276      | 0.055001538 | 0.057581366 | 0.05842356  | 0.058841386 |
| Pair (no Flush)   | 0.169411765     | 0.185037122 | 0.192486281 | 0.19491754  | 0.196123625 |
| Total Deals   | 22100           | 182104      | 1478256     | 5013320     | 11912160    |
| Suited 3 of a kind probability: 4 decks 0.000140706 6 decks: 0.000207447 8 decks: 0.000244456 |                 |             |             |             |             |
| Results for House Edge:   |                 |             |             |             |             |
| 1 Deck A: 2.50%   | 1 Deck B: 4.49% | 6 1 De      | ck C: 7.10% | 1 Deck D:   | 6.19%       |

| 4 Deck A: 3.08%             |                                    |                 |                 |        |
|-----------------------------|------------------------------------|-----------------|-----------------|--------|
| 6 Deck A: 4.36%             | 6 Deck B: 3.50%                    | 6 Deck C: 3.94% |                 |        |
| 8 Deck A: 3.66%<br>E: 3.29% | 8 Deck B: 2.80%<br>8 Deck F: 4.12% | 8 Deck C: 3.83% | 8 Deck D: 3.01% | 8 Deck |

An alternate version of the game can be played, called "Blackjack Twist", where only one blackjack hand is dealt to each player and the table layout only allows one side wager per player, has the same mathematics as Double Twist Blackjack, because the two bets the player may make in Double Twist Blackjack are treated independently of each other.

## **APPENDIX: 4-Card Pay Tables for Double Twist Blackjack**

Double Twist Blackjack can also be played using only the 4 initial cards from the player's two Blackjack hands, or using the initial two cards from a player hand combined with either the dealer's first two cards or the dealer's upcard and a special community card which is dealt if any players have made the side wager. In the second variation, bets cannot be settled until after players have completed their hands and the dealer's upcard has been turned over. In the third variation, the dealer must deal an additional face-up "community card" after dealing his own cards, if any players have made the 4-card side bet.

Nine pay tables were analyzed, where all hands containing a pair of Sixes or better are winners. Four of these involve a fixed payout for the top hand of "4 of a kind suited", and the other five involve a Progressive Jackpot payout for that top hand.

The calculated probabilities for all 4-card hands are as follows:

| Probabilities       | 6 decks   | 8 decks   |
|---------------------|-----------|-----------|
| 4 of a kind suited  | 0.0000020 | 0.0000030 |
| 4 card royal flush  | 0.0000134 | 0.0000133 |
| straight flush      | 0.0001339 | 0.0001332 |
| 4 of a kind         | 0.0003547 | 0.0003771 |
| straight            | 0.0092763 | 0.0092315 |
| 2 pair flush        | 0.0001813 | 0.0001989 |
| 3 of a kind flush   | 0.0001933 | 0.0002273 |
| 1 pair flush        | 0.0047854 | 0.0050004 |
| flush               | 0.0094235 | 0.0093780 |
| 2 pair              | 0.0151610 | 0.0154030 |
| 3 of a kind         | 0.0193736 | 0.0199042 |
| High Pair (6's-A's) | 0.2405209 | 0.2418325 |
| Low Pair (2's-5's)  | 0.1068982 | 0.1074811 |
| no pair             | 0.5936826 | 0.5908164 |
| Total               | 1         | 1         |

The following results were obtained (all payouts are on a "to 1" basis: pairs of Sixes through Aces are paid even money and pairs of Deuces through Fives lose):

Non-progressive Pay Tables:

|                    | 6 Decks- | 8 Decks- 6 Decks- |     | 8 Decks- |
|--------------------|----------|-------------------|-----|----------|
|                    | А        | А                 | В   | В        |
| 4 of a kind suited | 1000     | 1000              | 500 | 1000     |
| 4 card royal flush | 500      | 500               | 250 | 250      |
| straight flush     | 200      | 200               | 100 | 100      |
| 4 of a kind        | 50       | 50                | 50  | 50       |

| straight            | 20    | 10    | 20    | 15    |
|---------------------|-------|-------|-------|-------|
| flush               | 5     | 10    | 4     | 10    |
| 2 pair              | 4     | 4     | 3     | 3     |
| 3 of a kind         | 3     | 3     | 2     | 2     |
| High Pair (6's-A's) | 1     | 1     | 1     | 1     |
| Low Pair (2's-5's)  | -1    | -1    | -1    | -1    |
| no pair             | -1    | -1    | -1    | -1    |
| House Edge          | 2.96% | 3.97% | 9.65% | 4.55% |

Progressive Pay Tables:

|                          | 6 Decks- | 6 Decks- | 8 Decks- | 8 Decks- | 8 Decks- |
|--------------------------|----------|----------|----------|----------|----------|
|                          | С        | D        | С        | D        | E        |
| 4 of a kind suited       | 100%     | 100%     | 100%     | 100%     | 100%     |
| 4 card royal flush       | 1000     | 1000     | 500      | 1000     | 1000     |
| straight flush           | 100      | 100      | 100      | 100      | 100      |
| 4 of a kind              | 100      | 50       | 50       | 50       | 50       |
| straight                 | 10       | 10       | 10       | 10       | 10       |
| flush                    | 5        | 5        | 5        | 5        | 5        |
| 2 pair                   | 4        | 3        | 4        | 4        | 4        |
| 3 of a kind              | 2        | 2        | 2        | 2        | 3        |
| High Pair (6's-A's)      | 1        | 1        | 1        | 1        | 1        |
| Low Pair (2's-5's)       | -1       | -1       | -1       | -1       | -1       |
| no pair                  | -1       | -1       | -1       | -1       | -1       |
| Jackpot Seed Value       | 10000    | 10000    | 5000     | 10000    | 10000    |
| Progressive Contribution | 5.00%    | 5.00%    | 5.00%    | 5.00%    | 5.00%    |
| House Edge               | 6.26%    | 9.55%    | 8.51%    | 6.36%    | 4.37%    |
|                          |          |          |          |          |          |

The Jackpot Seed Value shown is denominated in betting units. When the minimum bet is \$5, a seed value of 10000 units will reset to \$50,000, and so on.